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The Market for Reputations as an Incentive Mechanism

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Reputational career concerns provide incentives for short-lived agents to work hard, but it is well known that these incentives disappear as an agent reaches retirement. This paper investigates the effects of a market for firm reputations on the life cycle incentives of firm owners to exert effort. A dynamic general equilibrium model with moral hazard and adverse selection generates two main results. First, incentives of young and old agents are quantitatively equal, implying that incentives are “ageless” with a market for reputations. Second, good reputations cannot act as effective sorting devices: in equilibrium, more able agents cannot outbid lesser ones in the market for good reputations. In addition, welfare analysis shows that social surplus can fall if clients observe trade in firm reputations.

I. Introduction

The effects of current performance on future payoffs are central to the economics of reputation. Fama (1980) argued that a competitive market for managerial labor will alleviate moral hazard and discipline managers to work. His argument claimed that future wages will depend on past performance, and hence managers will want to perform well. Holmström (1999) further developed this argument and showed that it has

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important life cycle implications: career concerns may be too strong early on and will disappear toward the end of one’s horizon. It is therefore arguable that if reputation concerns can be extended beyond an agent’s active career, then the problem of declining incentives can be mitigated.1

Indeed, there is a fundamental difference between an individual’s reputation and a firm’s reputation: a firm’s reputation is a tradable asset. This paper investigates the conditions that guarantee long-term incentives through an active market for reputations.2 In particular, this paper shows that if a firm’s name, or entity, is separated from its owner’s identity, then incentives can survive throughout the owner’s career.

Kreps (1990) first demonstrated that reputation can become a tradable asset—and provide incentives—even when agents live for only one period. The argument is simple: an agent will be trusted (and earn a premium) only if he acquires the good name of his predecessor and will be able to sell his own good name only if he himself honors trust. If the loss from running down a good name outweighs the benefits from abusive behavior, then agents will have incentives to honor trust. This behavior is supported by having a high price for a good name, so that the loss of not being able to sell a good name outweighs the gains from abusive behavior.

The appealing feature of Kreps’s equilibrium is that short-lived agents become “ageless”: they do not act myopically. The theory is problematic, however, because of multiple equilibria. In particular, many equilibria are supported in which reputation is meaningless and no incentives are provided. These “bad” equilibria are no more or less likely than the “good” reputational equilibrium. Furthermore, the good equilibrium is supported by clients’ response to previous behavior that is irrelevant to future fundamentals, since past behavior has no direct link to future performance, but is rather indirectly linked through the players’ strategies. Indeed, if one restricts beliefs and actions to depend only on payoff-relevant information, then only the nonreputational equilibrium survives. More important, this theory is mute with respect to how reputations arise and become valuable assets. Namely, there is no account of how a firm’s reputation, represented by the value of its name, varies in value, as is commonly observed in reality. This too is due to the fact that client beliefs and the value of a good name are not tied down.

This paper considers an economy in which overlapping generations

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1 In a strategic setting of repeated games with incomplete information, Kreps et al. (1982) also show that incentives to act in a “good” manner will be generated through reputational concerns. In their model too, incentives that support good behavior disappear as agents approach the end of the game.

2 Another mechanism that can mitigate end-of-career effects may be the legacy of a family name. This paper concentrates on a market approach without dynastic families.
of agents supply services to clients for two periods and then retire. Some agents are inherently good, whereas others choose how good to be at a private cost. Clients do not know the agent’s type and thus are exposed to both moral hazard (hidden action) and adverse selection (hidden information). Furthermore, clients do not observe the actual identities of the agents running the firms that provide them with services. This central assumption creates a separation of entity from identity: the intangible name of a firm can be traded across agents without clients observing this trade.

The paper’s first main insight provides a rationale for an active market for firm reputations, which is precisely the separation of entity from identity. If clients cannot observe trade in names, then good histories command a premium over no histories, which in turn causes good names to have value and to be traded.

The second main insight is that the market for names, which is supported by the presence of adverse selection, can alleviate the problems associated with moral hazard even with short-lived agents. Unlike previous reputation models in which incentives decline with age, here reputation concerns provide incentives for agents throughout their career: young agents are concerned with their future income from providing services, whereas old agents are concerned with their future income from selling their firm’s name. The incentives provided by these two mechanisms are quantitatively the same: good names are scarce, and their price will capture the benefits from having such a name. Thus the “ageless” feature of Kreps’s equilibrium arises endogenously in the present model.

These two insights depend on the assumption that clients cannot observe trade of names. If clients can observe trade, they can believe that only incompetent agents would buy a name rather than “build” one. As a result, providing clients with information about name trades can cause the market for names to fail and reduce social surplus. In some cases, however, the identity of new owners is public information. This, for example, is pertinent to medical practices in which new doctors are known to clients. The possible limitations of the insights above, and ways to accommodate them into a more general theory of reputation, are discussed in Section VII.

A third result is that the market for names cannot separate between more and less able agents. In particular, there is no equilibrium in which good agents fully separate themselves by buying successful names. Intuitively, if only good agents buy successful names, then clients cannot update beliefs downward when these names perform poorly. Hence, bad agents will value successful names more than good agents because their alternative option of starting their own successful name is bleak. A direct implication of this result is that the model generates sensible
reputation dynamics: reputations increase after good performance and decrease after bad performance, a crucial characteristic for reputations to provide incentives.

An important difference between this paper and other reputation models is the market equilibrium approach employed here, which closes the model with respect to determining the value of a reputation. This is key in deriving two of the main results in this paper: that incentives are ageless and that the market for names cannot fully sort agents.

Standard repeated-game models do not reveal these insights because they offer no direct economic link between the market for services and the prices agents pay for names. When a market equilibrium analysis is employed, the value of having a good reputation is determined vis-à-vis the option of having no reputation in the market for services.

There is a small literature beyond Kreps (1990) that models firm reputation as a tradable asset. In Tadelis (1999), an overlapping generations model with adverse selection but without moral hazard is analyzed, and it generates trade in names and a similar no-sorting result. The lack of moral hazard, however, prevents the model from analyzing incentive provision and welfare, which are central to this paper. Fang (1998) introduces moral hazard into a model that is similar to Tadelis (1999). Fang shows that reputational concerns of selling a good reputation can overcome moral hazard, but the lack of a general equilibrium analysis prevents tying incentives of old and young agents. Mailath and Samuelson (2001) consider a different model in which a firm provides a service for clients, and the observed quality is a noisy signal of the firm’s actions (imperfect monitoring). They too do not analyze the nature of life cycle incentives.3

There is another literature that is concerned with providing incentives to older agents using an overlapping generations demography without trade of reputations. Crémer (1986) shows that agents with finite lifetimes can belong to an organization (or social norm) that is an infinite entity, and some cooperation can be supported in equilibrium. Alesina and Spear (1988) and Harrington (1992) show that short-lived politicians can choose “far-sighted” policies using a party that plays a role similar to that of Crémer’s organization. Another related but more distant literature was introduced by Becker and Stigler (1974) and developed further by Lazear (1979). In these models a sequence of increasing

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3 They do show that bad types are likely to value a very good reputation more than good types. Their partial equilibrium analysis requires an exogenous assumption that good types have a better outside option than bad types, which arises endogenously in this paper’s model. Another paper that combines moral hazard and adverse selection to generate interesting reputation dynamics is Diamond (1989), but there, reputations belong to individuals and are not traded.
wages guarantees that the future of a finitely lived agent’s career is valuable, even as he reaches the end of his career.

II. The Economy

Consider a model in which in each period risk-neutral agents sell goods (or offer services) to risk-neutral clients for that period only. An outcome is either a success (high-quality), which gives a client a payoff of one, or a failure (low-quality), which gives a client a payoff of zero. Clients face both adverse selection and moral hazard. In particular, there are two indistinguishable types of agents: good agents, or $G$ types, in proportion $\gamma$, and opportunistic agents, or $O$ types, in proportion $1 - \gamma$. A $G$ type succeeds with probability $P_e \in (0, 1)$, and an $O$ type can choose his probability of success by exerting effort $e \in [0, 1]$ at a private cost $c(e)$, where his probability of success is given by $P_e(e) = e P_e$. Moral hazard is captured by assuming that effort is costly, $c'(e) > 0$. For convenience, assume that $c'()$ is twice continuously differentiable and that $c''(e) > 0$ (to ensure a unique solution to the agent’s problem), and let $c(0) = 0$.

Assume that agents are active in the economy for two periods, after which they retire, and wealth is valuable for retirement. Agents live as overlapping generations in which the total size of the population and the distribution of types of agents are constant over time. In contrast, clients live for only one period and can observe the firms’ track records. As in Diamond (1989) and Tadelis (1999), the implication of this assumption is that a firm’s reputation, summarized by its past performance, is the only intertemporal linkage.

Each agent in this economy runs his own firm, which is represented by a name, and it is assumed that every firm has a unique and distinct name. An agent can either choose a new name to represent his firm (which implies that he will have no track record) or buy a name from an agent who is about to retire, thus inheriting the track record associated with that name. The value of a firm’s service is determined by the perfect observation of that firm’s past performance and by the beliefs of clients that such past performance generates.

There is a continuum of clients and agents, and the price of supplying a service is determined competitively. To simplify, assume that the clients are on the long side of the market (a larger measure) so that each client pays her expected surplus when transacting with an agent. The results are robust to any division that gives the agents some positive surplus.

Assumption 1. The transaction’s outcome is not contractible.

A previous version of this paper had a third “bad” type that did not succeed. The results in that case are the same, and therefore the current formalization is more parsimonious. The previous version also had an extended section on welfare analysis that was dropped (see Tadelis 2001).
That is, problems of (court) verifiability prevent the parties from writing outcome-contingent contracts that depend on the realization of the outcome. This standard assumption implies that each client who employs an agent will pay up front the whole expected value of the service supplied.

Assumption 2. Shifts of name ownership are not observable by clients.

This means that the actual identity of the agent who provides the service is separated from the firm’s entity, that is, the name. This turns out to be a key property that introduces important noise into the economy: the impact of the current owner on the firm’s past performance is uncertain. In reality, shifts of ownership are often obscure. Of course, at some cost almost everything is observable, but to make the point, assumption 2 considers the extreme case of infinite costs of observation. This extreme assumption can be weakened to accommodate a situation in which only part of the population is ignorant to changes of ownership, in which case some qualitative results would carry over. This is further discussed in Sections V and VII.

Assumption 3. At the beginning of each period, every active agent can either choose to retain his past name or unobservably change it.

This assumption is symmetric to assumption 2 and allows a complete separation of a client’s identity from his firm. Once an agent chooses a new name, his past performance is erased and he can just as well be an agent who has now arrived into the economy with a clean record. It is reasonable to allow an agent to abandon his name and buy a name from another agent. In equilibrium, however, it turns out that agents who wish to abandon their past are indifferent between choosing a new name and buying a name. Therefore, assume without loss that agents who choose to erase their past will also choose a new name.

Assumption 4. An arbitrarily small independently and identically distributed (i.i.d.) measure $\epsilon \geq 0$ of agents cannot change their name.

This assumption eliminates some “unreasonable belief” equilibria, as will be described in Section III, and plays a role similar to that of “trembles” for refining unreasonable beliefs in extensive form games. The trembles selected are not “special” in that they uniquely generate the type of equilibria that are analyzed. Any combination of trembles that involves some “stickiness” of names will suffice to weed out equilibria with unreasonable beliefs. Assumptions 3 and 4 together can be thought of as a reduced form of a more realistic process of name changing (this is discussed in Tadelis [1999]). To save on notation, I shall take $\epsilon = 0$; for example, an infinite but countable number of agents from each type cannot change their name. The sequence of events in each period is illustrated in figure 1.

5 See the proof of proposition 2 and, in particular, n. 17.
III. Benchmark: No Market for Reputations

This section outlines a benchmark model that relates this paper to previous models in the reputation literature. The analysis of an economy without a market for reputations will also provide a useful benchmark for the welfare analysis of Section V.

In what follows, restrict attention to two periods, each with the sequence of events depicted in figure 1. To capture an overlapping generations demography, there will be three generations: generation 0 lives in the first period, generation 1 lives in both periods, and generation 2 lives in the second period. Furthermore, the size (measure) of each generation is equal. Thus this economy will always consist of a proportion $\gamma$ of $G$ types and a proportion $1 - \gamma$ of $O$ types. These demographics are described in figure 2.

It is assumed throughout the benchmark analysis that names cannot be traded. This implies that all generation 2 agents will have new names at the beginning of period 2. Agents of generation 1 will have the choice of sticking to their name or changing it, and agents of generation 0 will retire, and their firms' names will cease to exist.

The solution concept is a rational expectations equilibrium, which naturally applies to the model. In particular, at the beginning of $t = 1$, all agents have no history, and the wage $w_1$ will depend on clients' beliefs about the effort level of $O$ types. Since only past histories are observable, assume that two names with the same history generate the same expectations. This implies that at the beginning of $t = 2$ there will be history-dependent wages conditional on whether an agent had a past success ($S$), a failure ($F$), or a new name ($N$). Denote these wages by $w_2(h)$, $h \in \{S, F, N\}$.

A first obvious fact is that generation 1 agents who failed in period 1 will be better off changing their names and disguising themselves as the new agents. This is easily verified using Bayes' rule: if clients believed that names were not changed after a failure, then an agent with a past failure must be, on average, “worse” than an agent with a new name. As a result, and from assumptions 3 and 4, in equilibrium, only $S$ and

Fig. 1.—Time line for each period
Fig. 2.—Two-period economy

$N$ histories are observed with positive probability in period $t = 2$ (there will be a measure $\epsilon = 0$ of $F$ names). A second obvious fact is that $O$ types will choose $\epsilon = 0$ in their last active period since they have no future to look forward to and their wages are paid up front. This implies that $w_o(S)$ and $w_o(N)$ depend only on the clients’ beliefs about a $G$ type’s likelihood of having such a history. Let $\Pr[G|h]$ denote the conditional probability that an agent is a $G$ type given history $h$.

Assume without loss that second-period income is not discounted, so that the expected lifetime utility of a $G$ type at $t = 1$ is

$$Eu_G = w_1 + P_G w_2(S) + (1 - P_G) w_2(N),$$

and the expected utility of an $O$ type depends on his choice of effort $\epsilon$, yielding

$$Eu_O = w_1 + \epsilon P_G w_2(S) + (1 - \epsilon P_G) w_2(N) - c(\epsilon). \quad (1)$$

The equilibrium effort choice of $O$ types will affect second-period wages, and these in turn feed back into the incentives of $O$ types. Thus an equilibrium will be characterized by a tuple $(\epsilon, w_0, w_o(S), w_o(N), w_o(F))$, where $\epsilon$ is a best response given wages, and wages are correct given $\epsilon$. With correct beliefs about $\epsilon$,

$$w_1 = [\gamma + \frac{1}{2}(1 - \gamma)\epsilon] P_o \quad (2)$$

because all $G$ types of generations 0 and 1 will succeed with probability $P_o$ and only half the $O$ types (generation 1) will succeed with probability $\epsilon P_G$ (recall that generation 0 $O$ types exert no effort). As mentioned earlier, what determine second-period wages are the correct beliefs of a $G$ type behind any history, and they are calculated by applying Bayes’
rule. In particular, given an effort level \( e \in [0, 1] \), equilibrium beliefs imply that

\[
\Pr \{ G|S \} = \frac{\gamma P_G}{\gamma P_G + (1 - \gamma) e P'_G}
\]

and

\[
\Pr \{ G|N \} = \frac{\gamma(1 - P'_G) + \gamma}{\gamma(1 - P'_G) + (1 - \gamma)(1 - e P'_G) + 1}.
\]

That is, a firm with a past success is generated by \( G \) types and \( O \) types from generation 1 who succeeded, which accounts for (3) above. A firm with a new name is generated by all new (generation 2) agents and by all the agents from generation 1 who failed (and then changed their names), which accounts for (4) above. Given the equilibrium beliefs in (3) and (4), equilibrium wages are calculated by the equation (recall that clients value success at one and failure at zero).

The wage differential, \( \Delta w \equiv w_S - w_N \), provides incentives for young \( O \) types of generation 1. The effort level that maximizes their expected utility given in (1) above solves the first-order condition, \( P_G \Delta w = c'(e) \). Note that the equilibrium level of effort will be suboptimal since optimal effort must solve \( P_G = c'(e) \). It is also easy to see that \( \Delta w \) decreases in \( e \) (from [3] and [4] above), so that if \( O \) types choose higher effort in equilibrium, then the wage differential is smaller. Thus we can calculate the highest wage differential, \( \Delta w_H \), when \( O \) types choose \( e = 0 \), and the lowest wage differential, \( \Delta w_L \), when \( O \) types choose \( e = 1 \). These differentials are given by

\[
\Delta w_H = \frac{2(1 - \gamma)P_G}{2 - \gamma P_G}
\]

and \( \Delta w_L = 0 \).

**Proposition 1.** There exists a unique equilibrium. If \( P_G \Delta w_H \leq c'(0) \), then \( e = 0 \); if \( P_G \Delta w_H > c'(0) \), then \( e \in (0, 1) \).

The proofs of all the results are in the appendices. The intuition for proposition 1 is illustrated in figure 3. If \( e = 0 \) in the first period, then success must be attributed to good types, and the wage differential premium \( (P_G \Delta w_H) \) is greatest. This can be an equilibrium only if the marginal cost of effort at \( e = 0 \) is too high, which is the condition \( P_G \Delta w_H \leq c'(0) \). In figure 3 this occurs at the equilibrium point \( A \), for the cost of effort function \( c'_0 \). If this condition is violated, then the \( O \) types

\[\text{This follows immediately from maximizing social surplus: } dP_G - c(e), ~ e \in [0, 1]. \text{ Since in equilibrium } 0 < w(N) < w(S) < 1, \Delta w < 1 \text{ and suboptimal effort arises in equilibrium.}\]
must exert some effort in equilibrium, and since $c'(e)$ increases in $e$ and $\Delta w$ decreases in $e$, there must be a unique equilibrium. Points $B$ and $C$ depict two such equilibria for cost functions: $c_B(e)$ and $c_C(e)$, respectively.\footnote{In a discrete model in which $O$ types can be good ($e = 1$) or bad ($e = 0$), $e \in (0, 1)$ is like a mixed-strategy equilibrium. This would be similar to Kreps et al. (1982) and Diamond (1989).}

In summary, career concerns help solve the moral hazard problem for young agents, but not for old agents. This will provide a useful benchmark to analyze the effects of a market for reputations on the moral hazard problem of old and young agents.

IV. The Market for Reputations

This section describes the economic forces that cause reputations—captured by the firms’ names—to be traded in equilibrium. The analysis continues with a two-period model as shown in figure 2. This demonstrates that the results are independent of the economy’s horizon.
length, as long as the flavor of overlapping generations is maintained together with the model’s assumptions.

As before, clients will pay firms up front for their services given their (correct) beliefs about the composition of agents’ types for each name and the actions of O types. At the beginning of $t = 1$, agents from generations 0 and 1 choose names for their firms. Since no prior information is available to the clients, they will pay the same wage to all firms (as in the benchmark model), which depends on the behavior of O types in $t = 1$. At date $t = 2$, there is more going on. As in the benchmark model, there will be two kinds of firms: with and without a past history. In contrast, however, firms with a past history of success can be operated either by a continuing agent who succeeded and did not change his name or by a new agent who bought the name from a retiring agent.

An equilibrium will be characterized by the wages that clients pay to agents (firms) at $t = 1$, by the strategies of O types of each generation in every period, and by prices in two markets at $t = 2$: the wages clients pay to firms with different track records and the prices agents from generation 2 will pay for names with different track records that belonged to agents from generation 0. As before, since only past histories are observable, assume that two names with the same history generate the same expectations. Notice that it is assumed that only retiring agents from generation 0 can sell their names, and only new agents from generation 2 can buy names. Equilibria can be constructed in which agents from generation 1 sell and buy names, but because of the indifference result established in lemma 1 below, these equilibria are equivalent to the set of equilibria identified in the paper (with a different distribution of rents). In addition, restrict attention to equilibria in which only S names are traded, so that all agents who fail in the first period will change their name.8 As before, denote wages at $t = 2$ by $w_t(h)$, $h \in \{S, N, F\}$, and let $v(S)$ denote the price of S names.

**Proposition 2.** S names must be traded in all equilibria.

The proof of this proposition shows that if clients believed that S names were not being traded, then these names would become valuable assets that will trigger trade. Intuitively, there is always a supply of S names at the beginning of $t = 2$ (at least from the good types of generation 0 who succeeded). Therefore, no trade of S names in equilibrium implies that having a past success must be worthless to agents in $t = 2$. This in turn implies that opportunistic agents in the first period have no incentive to exert effort because it generates no future benefits. But if $S$

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8 One can construct equilibria in which both F and S names are traded. The main point of this section is that S names must be traded as proposition 2 states, but F names need not be traded.
names are not traded, then successful histories must belong to good types of generation 1, who continue to the second period. This would create high expectations for future success, which means that new agents will be willing to buy these names and disguise themselves as good types, since trading names is not observable.

If clients observe name trades, then this proposition fails: assigning “pessimistic” beliefs to clients that only bad types buy names supports an equilibrium with no trade of names. Thus the lack of information regarding trade of names is a driving force that guarantees an active market for names. Whether or not name trading is socially beneficial needs to be determined by considering the effects of markets for names on incentives in equilibrium. The following result is helpful to characterize equilibria of the two-period model.

Lemma 1. In any equilibrium, all new agents will be indifferent between buying an S name and not buying one, and $v(S) = w_S(S) - w_S(N)$.

This follows because in any equilibrium, $w_S(h)$ depends only on clients’ beliefs, which do not depend on the outcome of the second period. Since this is the last period, all types have the same benefit from buying an S name. As the supply of S names is scarce, the price of an S name must be set to cause indifference, which is the only way to clear the market. This leads to the following result.

Proposition 3. In any equilibrium, all first-period O types have identical incentives and thus choose the same effort level.

The intuition is simple: since the price of a name will be $v(S) = \Delta w$ (lemma 1), the wage differential that concerns young O types of generation 1 is equal to the sales premium that concerns old O types from generation 0, causing incentives to be “ageless.” Thus the incentives provided by career concerns (the wage premium) are quantitatively identical to the incentives created by the market for names; incentives are independent of an agent’s future horizon. This striking result illuminates the merits of a market equilibrium analysis in which wages and prices of names are tied down together. Note that in the model agents are active for only two periods, but the economic intuition seems quite general. Namely, a history generates value because it creates an expected sequence of wages to its owner and an expectation of selling the realized future. If valuable histories are scarce, then the price they command should equal their added value. This in turn means that, regardless of the owner’s age, he is internalizing the future sequence of these values (a “no-arbitrage” condition).

9 Name trading can still be supported in equilibrium. If a small proportion of clients observe trade, their beliefs must correspond to the actual buyers of names. This will lower the value of names. But if the proportion of informed clients is not too large, then proposition 2 still holds. This is discussed further in Tadelis (1999).
To continue with the equilibrium analysis, notice that the only endogenous parameter that affects the first-period wage is the (correct) beliefs clients have about the actions of $O$ types in the first period. The first-period wage must satisfy
\[ w_1 = [\gamma + (1 - \gamma)\varepsilon] p_s \]
similar to (2) in the benchmark model, but now all $O$ types of generations 0 and 1 succeed with probability $\varepsilon p_s$, not only the young of generation 1.

In any rational expectations equilibrium, clients must have correct beliefs about the composition of new agents who buy names at $t = 2$. Let $m$ (respectively $r$) denote the proportion of good (respectively opportunistic) types who buy $S$ names at $t = 2$. An equilibrium for the two-period model will be a tuple $(\mu, \rho, \varepsilon, w_2, w_3(F), w_3(S), v(S))$. Note that the wages firms charge clients and the prices new agents are willing to pay for names will be generated by the correct beliefs about $(\mu, \rho, \varepsilon)$, which uniquely determine the other equilibrium parameters.

In equilibrium, $(\mu, \rho, \varepsilon)$ must satisfy market clearing,
\[ \gamma p_s + (1 - \gamma)\varepsilon p_s = \mu \gamma + \rho (1 - \gamma), \]
which guarantees that the supply of $S$ names (the left-hand side of [5]) is equal to the demand (the right-hand side of [5]). Recall that clients will pay their full expected surplus up front, so in equilibrium it must be that, for all $h,
\[ w(h) = \Pr\{G|F \} w_s(h) = \Pr\{G|F \} \cdot p_s \text{ (only G types succeed in } t = 2). \]

Given $(\mu, \rho, \varepsilon)$, by Bayes’ rule,
\[ \Pr\{G|S\} = \frac{\gamma p_s + \mu \gamma}{\gamma p_s + (1 - \gamma)\varepsilon p_s + \mu \gamma + \rho (1 - \gamma)} \]
\[ = \frac{\gamma p_s + \gamma \mu}{2\gamma p_s + 2(1 - \gamma)\varepsilon p_s} \] \hspace{1cm} (6)

and
\[ \Pr\{G|N\} = \frac{\gamma (1 - p_s) + (1 - \mu) \gamma}{\gamma (1 - p_s) + (1 - \gamma)(1 - \varepsilon p_s) + (1 - \mu) \gamma + (1 - \rho)(1 - \gamma)} \]
\[ = \frac{2\gamma - \gamma p_s - \mu \gamma}{2 - 2\gamma p_s - 2\varepsilon p_s (1 - \gamma)}, \] \hspace{1cm} (7)
where the second equality in both equations follows from market clearing and simple algebra. The following proposition characterizes the set of equilibria in which only $S$ names are traded.

**Proposition 4.** There exist $\underline{\mu} < \bar{\mu}$ so that $(\mu, \rho, \varepsilon)$ is an equilibrium if and only if the following three conditions hold: (i) $\mu \in [\underline{\mu}, \bar{\mu}]$; (ii)
(\(\mu, \rho, e\)) satisfy market clearing; and (iii) \(c'(e) \geq \Delta \bar{u}(e) P_\rho\) with equality if \(e \in (0, 1]\).

Proposition 4 implies that there is a continuum of equilibria with respect to prices: the interval \([\bar{\mu}, \hat{\mu}]\) is nonempty, and any \(\mu \in [\bar{\mu}, \hat{\mu}]\) can be supported in equilibrium. This follows from the indifference result of lemma 1, which established that the price for names must be equal to the wage differential they generate. The equilibrium price for \(S\) names is increasing in \(\mu\) (the proportion of \(G\) types who buy them) so that \(\mu = \bar{\mu}\) supports the lowest-price equilibrium. Depending on parameter values, the lowest price is either zero (many good types—high \(\gamma\)—implies that having no history is quite good) or positive (very few good types—low \(\gamma\)—implies that even if only \(O\) types buy \(S\) names, these names are still better than having no history). Any equilibrium with \(\mu > \bar{\mu}\) commands a positive price for \(S\) names.

The multiplicity of equilibria, which are all qualitatively similar, makes it difficult to analyze the effect of the market for names on the incentives of opportunistic agents in comparison to the benchmark model. A way to proceed is to identify a reasonable equilibrium selection and then compare the two models in which each has a unique equilibrium. Since the multiplicity arises because sellers who buy names are indifferent, it is reasonable to assume that the matching between buyers and sellers of names is random, which causes the composition of types of name buyers to be equal to the exogenous composition of types. This implies that \(\mu\) and \(\rho\) are given by

\[
\mu^* = \rho^* = \gamma P_\rho + (1 - \gamma) e P_\rho.
\]

This selection can be endogenized by adding a second dimension of agent heterogeneity that would break the indifference demonstrated in lemma 1 and choose the selection above as a unique equilibrium. For example, agents may vary with respect to their cost of purchasing a firm versus building a new one: when an agent creates his own firm, it is tailored to his specifications, whereas buying an existing enterprise may require some adaptations or modifications. This is explicitly modeled in Tadelis (2002).

To complete the characterization of equilibrium, the value of \(\mu^*\) will be simultaneously determined with the level of effort \(e\). Let \(\Delta w_\rho(\mu^*)\) be

10 Formally, let \(\pi \in [0, \hat{\pi}]\) be the extra cost associated with purchasing an existing firm (name), and let \(\pi\) be i.i.d. across all agents with the cumulative distribution function \(G(\cdot)\) and with positive density \(g(\cdot)\) over the domain \([0, \hat{\pi}]\). An agent with cost \(\hat{\pi}\) will buy an \(S\) name only if it is worthwhile given his costs, that is, only if \(v(S) \leq w_0(S) - w_0(N) - \hat{\pi}\). This implies that there will exist some \(\pi^* \in [0, \hat{\pi}]\) such that all agents with cost \(\pi < \pi^*\) will buy \(S\) names, and other agents will not, independent of their type. It is therefore convenient to consider the unique equilibrium derived from the limit \(\pi^* \to 0\). This can, e.g., follow from \(G(\cdot)\) being uniform on \([0, \hat{\pi}]\) and letting \(\hat{\pi} \to 0\). The i.i.d. assumption guarantees that \(\mu^* = \rho^*\).
the high-wage differential in the unique equilibrium selected above. If \( c'(0) \geq \Delta w_t(\mu^*) P_c \), then in equilibrium we must have \( e = 0 \). If \( c'(0) < \Delta w_t(\mu^*) P_c \), then the unique equilibrium has \( e \in (0, 1) \), derived from \( c'(e) = \Delta w(\mu^*) P_c \). From (6) and (7), it is straightforward to compute the high- and low-wage differentials:

\[
\Delta w_T(\mu^*) = \frac{P_c(1 - \gamma)}{2(1 - \gamma P_c)}
\]

and \( \Delta w_L(\mu^*) = 0 \).

V. Incentive Effects and Welfare Analysis

As proposition 2 implies, if the identity of new owners is public information, then pessimistic beliefs will cause the market for names to shut down, resulting in an economy with no trade of names. By comparing the models in Sections III and IV (without and with a market for names), one can perform welfare comparisons.

Compared to the benchmark model, name trading will have two effects: First, for the agents of generation 0, it provides a potential incentive to exert effort in their terminal period. Second, the incentives for the agents of generation 1 in their initial period may be changed, thus affecting their career concerns. This second effect on “young” agents is ambiguous: it may be that the introduction of a market for names will cause the wage differential to decrease, thus lowering their incentives.

To see how parameters affect the welfare conclusions, it is again illustrative to consider the high-wage differential. If \( \Delta w_t(\mu^*) < \Delta w_p \), then there exist cost functions \( c() \) for which \( P_c \Delta w_t(\mu^*) < c'(0) < P_c \Delta w_p \). In this case, young agents exert some effort in the benchmark model. But with trade of names, neither young nor old agents will exert effort, causing a decrease in social surplus. Clearly, if \( \Delta w_t(\mu^*) > \Delta w_p \), then the opposite is true. This implies that when name trading becomes public information, the economy can move from an equilibrium with effort to an equilibrium without effort only if \( \Delta w_t(\mu^*) > \Delta w_p \), which reduces to \( P_c > 2/3\gamma \). If, however, standard Inada conditions are imposed on \( c() \), then in equilibrium \( e \in (0, 1) \). From the first-order condition of the benchmark model,

\[
c'(e^{BM}) = \frac{2\gamma P_c^2 [1 - \gamma - (1 - \gamma)e^{BM}]}{[\gamma + (1 - \gamma)e^{BM}][2 - \gamma P_c - (1 - \gamma)e^{BM}]}.
\]
whereas the first-order condition for the model with a market for names is

\[ c'(e^*) = \frac{\gamma P^*_G[1 - \gamma - (1 - \gamma)e^*]}{2[\gamma + (1 - \gamma)e^*][1 - \gamma P^*_G - (1 - \gamma)e^*P^*_G]} \tag{9} \]

From the previous analysis, without trade, all \( O \) types of generation 0 do not exert effort, whereas with trade they do. This means that with trade there are twice as many agents exerting effort, but this effort level \( e^* \) may be smaller than \( e^{BM} \). Since social surplus is additive, given \( e^{BM} \) and \( e^* \) from (8) and (9), trade of names is socially beneficial if and only if \( \frac{1}{2}[e^* - c(e^*)] > e^{BM} - c(e^{BM}) \). By choosing a specific cost function \( c(\cdot) \), one can compute the social surplus for both models over the parameter space \( (\gamma, P^*_G) \in [0, 1] \times [0, 1] \). For the quadratic example, \( c(e) = e^2/2 \), figure 4a illustrates that \( \frac{1}{2}[e^* - c(e^*)] - [e^{BM} - c(e^{BM})] > 0 \) over the whole range of parameters, implying that trading names always increases social surplus. For the exponential example, \( c(e) = \frac{1}{10} \cdot \exp(e) - \frac{1}{10} \), figure 4b illustrates that for high values of \( P^*_G \) trade is more efficient, whereas for low values of \( P^*_G \) no trade is more efficient.

I cannot claim that there are “reasonable” welfare conclusions as to whether trade of names is better or worse. Instead, it is interesting to realize that more information, that is, making name transfers observable, can cause the market for names to collapse and eliminate incentives for older agents. In many cases of the model, this can be detrimental for social surplus.

To more seriously consider young and old agents, the next section considers the infinite-horizon economy. It shows that the results obtained above are robust and derives an interesting no-sorting result.

VI. Longer Horizons and Reputational Sorting

Consider the infinite-horizon version of the model described above. In every period a new cohort of agents enters the economy and can buy names from the cohort that is retiring, so that the economy never terminates. After living for one period, agents can either continue with their name or change it; after two periods, they can sell their name. The following proposition parallels proposition 2 for the infinite-horizon model.

Proposition 5. Names consisting of only successes must be traded in all equilibria.

The proof of this proposition is omitted since it is almost identical to the proof of proposition 2. The intuition is the same: these names
Fig. 4.—Surplus difference: a, $c(\delta) = \frac{1}{2} \delta^2$; b, $c(\delta) = \frac{\exp(\delta)}{10} - \frac{1}{\pi}$
have a positive supply, and under the assumption of no trade, it must be that they convey positive information and therefore have value.\footnote{The proof that $NS$ names must be traded in every period (where $NS$ means that the name was created last period and had a success) is identical to the proof of proposition 2. An induction argument implies that all names with any consecutive number of successes (with no failures) must be traded.}

Since names are associated with histories, this introduces a rather demanding and complex problem for the analysis of the infinite-horizon model. Formally, the set of histories $H$ is the set of all finite- and infinite-length histories consisting of successes and failures (including $N$, no history). Then, there can possibly be an infinite number of histories with different reputation values.

A way to simplify the analysis is to distinguish between histories and reputations (see Tadelis 2002). Appendix B introduces a formal reduction of histories into equivalence classes using a well-defined reputation reduction. This reduction is rather straightforward, and it uses the model’s stationarity.

Using this reduction, Appendix B analyzes a stationary steady-state equilibrium (SSE) that is similar qualitatively to the equilibrium of the two-period model: names last as long as they do not fail; once they fail, they are rationally discarded. The wage differentials for the derived SSE are exactly those derived for the two-period model, but since there is no terminal period, incentives are provided in every period. Note that the equilibrium identified in the finite-horizon benchmark model is the unique stationary SSE for the infinite-horizon model without trade of names.\footnote{One can possibly construct cyclical equilibria for the model without trade of names. Intuitively, if in every odd period $O$ types work harder than in every even period, then the wage differential in odd periods is lower than that in even periods, which supports this type of cyclical behavior.}

An intuitive conjecture is that if the economy lasts for more than two periods, then the value of a good reputation should be higher for a good type who is more likely to maintain it than for an opportunistic type who has to exert effort to do so. This reasoning would be consistent with the theories of Klein and Leffler (1981) and Kreps (1990), if their ideas were incorporated into the current incomplete information framework. That is, if good types find it easier to maintain a reputation, then they should be able to outbid opportunistic types who are more likely to ruin a reputation. Notice, however, that in the equilibrium analyzed above, this separation did not occur. It turns out that this is no coincidence, as the following proposition states.

**Proposition 6.** For the infinite-horizon model, there is no equilibrium in which only $S$ names are traded, and they are bought only by good types and by opportunistic types who choose to be good.

To see this, consider a candidate sorting equilibrium in which only $S$ names are traded, and they are bought only by good types and by opportunistic types who choose to be good.
good types buy S names. This means that clients must (correctly) predict that any S name is bought by a good type, so that a name with a history that starts with a success is more likely to belong to a good type no matter what the continuation history will be. This may seem somewhat counter-intuitive, but the intuition is quite simple: If a name had a success in period t, then the person who continues with this name at period t + 1 is more likely to be good. Thus failures will not be “punished” with low wages since clients cannot update their beliefs too strongly, and the value of the name will remain high.

This, in turn, causes O types to value an S name more than G types because the value of buying an S name depends on the alternative option of not buying one. The O types face a less attractive future if they start with a new name because it is harder for them to build a good reputation. Therefore, if the stream of payments from having an S name is the same (or close) for both types, then having a poor alternative will make O types value an S name more than G types. Thus, in any equilibrium, enough O types (choosing \( e < 1 \)) buy S names so that clients will sufficiently update their beliefs after a name with a good reputation starts failing.

In Tadelis (1999), two reputational effects arise in a pure adverse selection model. The reputation maintenance effect captures the idea that more able types are more likely to maintain a good name. This allows good types to reap benefits over a longer horizon (on average), which in turn gives them a higher willingness to pay for a good name. The reputation start-up effect captures the idea that good types can more easily build a good name. Therefore, if a firm’s reputation does not depreciate, then good types will have a lower willingness to pay than bad types. The introduction of moral hazard in this paper shows the generality of these effects. It is the market equilibrium approach of both models that identifies the alternative of buying a good name, which is building one.13

VII. Concluding Remarks

A. Summary

This paper suggests that if firm names can change hands without clients’ awareness, then clients must constantly update their beliefs about the type of agent running the firm. Furthermore, their updating must follow

13 With terminology from the signaling literature, the “single-crossing” condition in my model is endogenous and depends on who buys names. Mailath and Samuelson (2001) develop a repeated-game model with incomplete information and imperfect monitoring and show that when a reputation is “too good,” it is more likely to be bought by a bad type. Given their partial equilibrium approach, however, they need to exogenously assume that good types have a better outside option than bad types. In my models the outside option is derived endogenously using a market equilibrium analysis.
a sensible rule: good performance causes higher expectations, whereas bad performance causes lower expectations. These dynamics cause good reputations to have value, which in turn gives agents incentives to maintain a good reputation throughout their career.

An important difference between the model in this paper and standard models of reputation is in the market equilibrium approach employed here. This is key in deriving two main results. First, young and old agents face the same incentives created by the market. The markets for services and for names are linked: good names are scarce, and their price captures their full value—the wage differential they generate in the market for services. Second, reputations cannot fully sort good agents from opportunistic ones. Models of reputation that use a partial equilibrium repeated-games approach, such as Klein and Leffler (1981) or Kreps (1990), show that good reputations support good behavior. One might then conclude that good reputations will be valued more by agents who intend to be good, either by characteristic (type) or by choice (action). A market equilibrium analysis, however, shows that such separation is impossible. The value of a reputation depends on the updating of clients’ beliefs, which depends on the types of agents that buy reputations. If “too many” of the agents buying good names are expected to perform well, then failure causes weak updating of client beliefs. This in turn causes good reputations to be valued more by agents who are less competent, because the alternative of starting with a clean record is bad for them.

B. Extensions

This paper is an attempt to model, and understand, a possible mechanism that provides incentives for older agents and contributes to the literature on life cycle incentives. The central assumption, that clients do not observe transfers of ownership, is not reasonable for all industries (e.g., medical practices). An owner of a good reputation (a successful clientele) would very much like to announce that he is selling his name to a good type and then get a higher price for the firm’s good name. But proposition 6 implies that this is impossible: there is no credible way that an owner could commit to selling to a good type, even if he can distinguish between good and opportunistic types and clients can observe trade.14

14 From proposition 4, if more \(G\) types buy a name, then the price of that name is higher, so the name’s initial owner would benefit from having better agents buy names in equilibrium. Interestingly, even when ownership shifts are observable, the no-sorting result of proposition 6 still holds: in equilibria with trade of names, if beliefs are “too good,” then updating is too weak to support separation, and owners of names cannot commit to this strategy.
Indeed, it is realistic to assume that experienced medical doctors, lawyers, or accountants have some ability to identify and screen the young professionals who can take over their practice. In this case, the senior owners would want to commit to selling their firm to competent successors. One way to implement this is to employ young agents in a sort of “apprentice relationship” and make this observable to their clients. Then, after clients observe the good outcome generated by the young professional, their beliefs are more secure, and the senior owner can sell the firm (the clientele) at a higher price. Screening would be an equilibrium if bad outcomes would cause a loss of reputation, which directly harms the senior owner.

Notice that this story is different from the model of the paper in several important ways, primarily the ability to screen young professionals and the ability of clients to observe these apprentice relationships and trade of ownership (or at least promotion to partner). By modeling these issues seriously and building on the insights of this paper, one may be able to develop an interesting theory of such relationships. This is left for future research.15

C. Empirical Relevance

The model of this paper is very stylized but seems to fit small owner-operated firms with transient clients, such as restaurants and small service businesses, but is harder to link to larger firms. Nonetheless, the insights may apply to more complex organizations and shed some light on incentive provision. For example, the results suggest that key figures in an organization should have a stake in the organization’s future reputation. Gibbons and Murphy (1992) show that the loss of career concerns of managers close to retirement can be supplemented by explicit contracts. They support their theoretical results with data, yet the strength of explicit incentives observed (a share of current profit) is remarkably low. Since future, and not current, prospects are at the heart of reputational incentives, it would be wise to compensate older managers with vested stocks/options. In a recent article, Murphy (1999) indicates that top executive compensation has a large component of long-term vested stock options. This idea complements Fama’s (1980) argument: competitive forces in the managerial labor market alleviate moral hazard, but there must be a stake in the firm’s future, beyond a manager’s finite career. Some of the value of a firm’s name should be allocated to managerial labor. Notice that a version of the nonobserv-

15 Using an overlapping generations market equilibrium analysis, Tadelis and Rangel (2001) analyze an apprenticeship model in which a young agent’s human capital is increased by working with an old agent. Rather than focusing on reputation, the model focuses on the sorting and efficiency properties of competitive signaling in equilibrium.
ability assumption 2 is not implausible. Even if analysts and investors see such changes in management, it is enough that clients of the firm’s product are oblivious to these staffing changes, since their beliefs about future performance are what generate value to the firm’s reputation.

In organizations with many agents, there may be a free-rider problem associated with maintaining a good name. In their seminal paper, Alchian and Demsetz (1972) raise an important question related to free-riding in an organization: “One method of reducing shirking is for someone to specialize as a monitor to check the input performance of team members. But who will monitor the monitor?” (pp. 781–82). They suggested that the monitor is provided with correct incentives when he is the residual claimant to the team’s profits. The argument is that market forces will cause the monitor to internalize the social costs and benefits of monitoring. But monitors do have finite careers, implying that monitors should lose incentives as their career comes to its end. This paper suggests that the residual claimant to a firm’s profits should also be the residual claimant to the value of its name, thus internalizing the full current and future value of his monitoring efforts.

D. Policy Relevance

Casual empiricism suggests that modern capitalist economies have legal systems that support the separation of entity from identity and facilitate well-functioning markets for names. Names are identified as proprietary assets with well-defined property rights. The analysis above suggests that without such property rights and markets for names, incentives are eroded as entrepreneurial agents approach retirement. Therefore, such legal systems are indeed beneficial from an efficiency perspective: the market for firms’ names is complementary to both product and labor markets in a well-functioning market economy.

With respect to small owner-operated (“mom and pop”) businesses, the analysis suggests that a regulatory action that makes the event of a name transfer public information can be socially harmful: the market for names can collapse, thereby destroying the endogenous incentives created by this market.16 This is a rather stark suggestion, and there are many caveats that lie in the assumptions of the model. For example, if there is a valuable “match” component between agents’ ability and clients’ needs, then identifying when a firm changes ownership may be very important to those types of clients.

On a final note, the analysis may also suggest that different tax dis-

16 Recall Hirshleifer’s (1971) seminal paper, which shows that acquisition of private information may destroy markets for efficient risk sharing. Here, risk sharing is irrelevant, but information disclosure may destroy markets that provide incentives.
tortions will cause distortions in the dynamics of career concerns. For example, with respect to such small owner-operated firms, young agents are subject to income taxes, whereas retiring agents will pay capital gains taxes from the sale of a successful name. Thus the relative magnitude of these two tax instruments can affect the dynamic allocation of effort. Similar implications can be relevant for the taxation of vested stock options. If the tax rates on these options differ from income taxes, this again can create a distortion in the lifetime career concerns of managers. Clearly, there are many other issues that are important for taxation, and this discussion raises one more possibility for consideration.

Appendix A
Proofs
This Appendix contains proofs of propositions 1–4 and lemma 1.

Proof of Proposition 1
If \( c'(0) < \Delta w_0 \) then \( e = 0 \) cannot be an equilibrium. Since \( \Delta w \) is continuous and decreasing in \( e \), \( c'(e) > 0 \), and at \( e = 1 \) we have \( \Delta w_0 = 0 \), there exists a unique \( e \in (0, 1) \) such that \( c'(e) = \Delta w_0 \) and this is the unique equilibrium. If, however, \( c'(0) \geq \Delta w_0 \), then when all \( O \) types choose \( e = 0 \), this is an equilibrium. No other equilibrium can be sustained since any higher effort level \( \hat{e} \) implies a lower \( \Delta w \) and \( c'(\hat{e}) > c'(0) \). Q.E.D.

Proof of Proposition 2
Assume in negation that there exists an equilibrium in which no names are traded. This implies that the value of an \( S \) name cannot be positive since there is a positive supply of \( S \) names equal to the measure \( \gamma p_i + (1 - \gamma) p_i > 0 \). Observe that \( S \) names have nonpositive value if and only if \( w_S(S) \leq w_S(N) \). Since names are not traded, it is a dominant strategy for \( O \) types of generation 0 to choose \( e = 0 \). Furthermore, \( w_S(S) \leq w_S(N) \) implies that \( e = 0 \) is a dominant strategy for \( O \) types of generation 1 in period \( t = 1 \), and therefore only \( G \) types will succeed. In this case, assumption 4 implies that \( \Pr(G|S) = 1 \), and \( 1 - \gamma > 0 \) implies that \( \Pr(G|N) < 1 \). This in turn implies that \( w_s(S) > w_s(N) \), a contradiction. Q.E.D.

Proof of Lemma 1
The only effect a name has for agents at \( t = 2 \) is to (weakly) increase their wages, and this effect is identical for all types since it does not depend on period 2 outcomes. Thus, if some agents prefer buying an \( S \) name, then all agents do.

---

Assumption 4 guarantees that \( \Pr(G|S) = 1 \) when no names are traded. This assumption rules out equilibria of the following form: All agents abandon their name after the first period, and in the second period, clients believe that a firm with any history is worse than the average firm with no history. Since a measure \( e \) of the agents will not be able to abandon their name, these beliefs cannot be sustained in equilibrium.
The measure of new agents is one, whereas the measure of supplied \( S \) names is \( [\gamma + \varepsilon(1 - \gamma)]p_1 < 1 \), which creates excess demand for any price \( v(S) < w_2(S) - w_3(N) \). It cannot be that \( v(S) > w_5(S) - w_3(N) \) since from proposition 1 trade of \( S \) names must occur, implying that \( v(S) = w_3(S) - w_4(N) \). Q.E.D.

**Proof of Proposition 3**

In equilibrium, \( O \) types of generation 1 set and \( O \) types of generation 0 set \( c(e) = v(S)p_o \) and from lemma 1, \( v(S) = w_3(S) - w_4(N) \). Q.E.D.

**Proof of Proposition 4**

Market clearing implies that \( (m, \rho, \varepsilon) \) must satisfy (5). Also, it must be the case that in equilibrium \( v(S) = w_3(S) - w_4(N) \geq 0 \). Since \( w_i(h) = \Pr[G|h] \cdot p_o \) from (6) and (7) above, \( w_3(S) - w_4(N) \geq 0 \) can be rewritten as

\[
\frac{\gamma p_o + \gamma \mu}{2\gamma p_o + 2(1 - \gamma) \mu} \geq \frac{2\gamma - \gamma p_o - \mu \gamma}{2 - 2\gamma p_o - 2\varepsilon p(1 - \gamma)},
\]

which after rearrangement becomes

\[
\mu \geq (2\gamma - 1)p_o + 2\varepsilon p(1 - \gamma).
\] (A1)

Denote the right-hand side of (A1) by \( \tilde{\mu} \). Observe that \( \tilde{\mu} \) is increasing in \( \varepsilon \), and from \( \varepsilon < 1 \) it follows that \( \tilde{\mu} < p_o \). Define \( \hat{\mu} = \max\{0, \tilde{\mu}\} \), and note that to satisfy \( v(S) \geq 0 \) it must be that \( \hat{\mu} \geq \tilde{\mu} \). Now, let \( \hat{\mu} \) be the proportion of \( G \) types that are needed to clear the market with no \( O \) types buying \( S \) names at \( t = 2 \), which is derived from market clearing as follows: \( \hat{\mu} = p_o\{1 + [\varepsilon(1 - 1)]\} \). Notice that \( \hat{\mu} \geq p_o \) since \( 1/\gamma \geq 1 \). Also, for certain parameter values (e.g., \( \gamma << 1 \)), we can have \( \hat{\mu} > 1 \), which means that only good types buying \( S \) names cannot clear the market. Define \( \hat{\mu} = \min\{\tilde{\mu}, 1\} \). The interval \([\hat{\mu}, \hat{\mu}]\) is nonempty, which follows from \( \hat{\mu} < p_o \leq \hat{\mu} \). Thus, if \( (\mu, \rho, \varepsilon) \) is an equilibrium, then conditions i–iii must be satisfied. The converse follows immediately. Q.E.D.

**Appendix B**

This Appendix provides an analysis of the infinite-horizon model and a proof of proposition 6.

**Infinite-Horizon Analysis**

Let \( H \) denote the set of all possible histories, and let \( h \in H \) be a generic history. To reduce the set \( H \) to a smaller set of reputations, consider four equivalence classes of histories. That is, clients will treat any two histories in the same equivalence class with the same beliefs. In particular, we have the following definition.
stationarity is used to obtain the market-clearing condition in their last period, regardless of what happened in their first period. Thus is whereas his utility from not buying a

benefit, there is a scarcity of names and the price of an

name is Since all agents—regardless of their type—get this

name is and the value from having an

name must be

never better than a new name. This implies that incentives do not depend on

the name an agent has since any failure will cause a change of name. Thus the

selection of random matching and consistent with restricting attention to the

case in which only S names are traded (it will be confirmed that F or M names

will not be traded, and agents with M or F names would prefer to change them
to N names). That is, the names from the equivalence class $H_t$ are randomly

assigned to the young agents who enter the economy. It will also be confirmed

that the restriction to these four classes is consistent with equilibrium beliefs,

so that no irrationality is involved.

Denote the SSE wages as $w(N)$, $w(S)$, $w(M)$, and $w(F)$, respectively. In each

period there are “young” and “old” agents as in any standard overlapping gen-

erations model. The following result is parallel to proposition 3.

**Proposition B1.** In any SSE, all young and old $O$ types have identical incen-
tives and thus choose the same effort level.

**Proof.** If in equilibrium only $S$ names are traded, then all other names are

never better than a new name. This implies that incentives do not depend on

the name an agent has since any failure will cause a change of name. Thus the

utility an agent $i$ who succeeds with probability $P_i$ gets from buying an $S$ name is

$u_i(S) = w(S) + P_i w(S) + (1 - P_i) w(N)$, whereas his utility from not buying a

name is $u_i(N) = w(N) + P_i w(S) + (1 - P_i) w(N)$, and the value from having an $S$

name is $\Delta u_i = w(S) - w(N)$. Since all agents—regardless of their type—get this

benefit, there is a scarcity of names and the price of an $S$ name must be

$w(S) = w(S) - w(N)$, Q.E.D.

It follows that the supply of $S$ names has the same structure as for the two-

period model because old agents who sell an $S$ name are those who succeeded

in their last period, regardless of what happened in their first period. Thus

stationarity is used to obtain the market-clearing condition

$$\gamma P_e + (1 - \gamma) e P_e = \mu^*.$$  \hspace{1cm} (B1)

The SSE is therefore characterized by the tuple $[\mu^*, e, w(S), w(N), w(F), w(M), \psi]$. Computing the SSE wages using Bayes’ rule as before yields

$$w(S) = \frac{\gamma (P_e + \mu^*) + e (1 - \gamma) (e P_e + \mu^*)}{\gamma P_e + (1 - \gamma) e P_e + \mu^*} \cdot P_e$$

$$= \frac{\gamma [P_e + \gamma P_e + (1 - \gamma) e P_e] + e (1 - \gamma) [e P_e + \gamma P_e + (1 - \gamma) e P_e]}{2[\gamma P_e + (1 - \gamma) e P_e]} \cdot P_e \hspace{1cm} (B2)$$
Market for reputations

\[ w(N) = \frac{\gamma(1 - P^* + 1 - \mu^*) + \epsilon(1 - \gamma)(1 - P^* + 1 - \mu^*)}{\gamma(1 - P^*) + (1 - \gamma)(1 - P^*) + 1 - \mu^*} \cdot P_c \]

\[ = \frac{\gamma[2 - \gamma P^* - (1 - \gamma) e P^* - P_c] + \epsilon(1 - \gamma)[2 - \gamma P^* - (1 - \gamma) e P^* - P_c]}{2[1 - \gamma P^* - (1 - \gamma) e P^*]} \cdot P_c \quad \text{(B3)} \]

where the second equality in both cases follows from substituting \( \mu^* \) from (B1) above. With Bayes' rule, an \( M \) name must be generated by an \( S \) name that was followed by a failure and then continues to be active. Such a name can be generated only by young agents who bought an \( S \) name and then failed and are stuck with their name. According to the equilibrium under construction, no matter how these names evolve (success or failure), they will not be sold; hence, once an opportunistic type continues with such a name, he will choose \( \epsilon = 0 \). Thus

\[ w(M) = \frac{\mu^* \gamma(1 - P^*)}{\mu^* \gamma(1 - P^*) + \epsilon(1 - \gamma)(1 - P^*) + \mu^*(1 - \gamma)(1 - \gamma)} \cdot P_c \]

\[ = \frac{\gamma(1 - P^*)}{1 - \gamma P^* - (1 - \gamma) e P^*} \cdot P_c \]

Similarly, in equilibrium, an \( F \) name is generated by an \( N \) name that was followed by a failure and then continues to be active. (From assumption 4, any other history in \( H_f \) should not be observed in equilibrium, which will be supported by the appropriate beliefs.) Such a name can be generated only by young agents who did not buy an \( S \) name and then failed and are stuck with their name. As with \( M \) names, opportunistic types who continue with such a name will choose \( \epsilon = 0 \). Thus

\[ w(F) = \frac{(1 - \mu^*) \gamma(1 - P^*)}{(1 - \mu^*) \gamma(1 - P^*) + \epsilon(1 - \gamma)(1 - P^*) + (1 - \mu^*)(1 - \gamma)(1 - \gamma)} \cdot P_c \]

\[ = \frac{\gamma(1 - P^*)}{1 - \gamma P^* - (1 - \gamma) e P^*} \cdot P_c \quad \text{(B4)} \]

Any other name that is considered to be an \( F \) name, \( h \in H_f \setminus NF \) (by the definition of the reputation reduction), occurs with zero probability, and it is possible to assign any beliefs by clients to any such name, in particular to have all such histories command a wage of \( w(F) \) as calculated above.

The analysis establishes that in our candidate equilibrium, \( w(F) = w(M) \), and to verify that an agent who failed will prefer to change names over sticking to
an F or M name, it must be shown that \( w(N) \geq w(F) \). Let \( \Delta = \text{Pr}(G|N) - \text{Pr}(G|F) \). Using (B3) and (B4) above, we have

\[
\Delta = \frac{\gamma(2 - \gamma P_e - \epsilon (1 - \gamma) P_e - P_e) + \epsilon (1 - \gamma)(2 - \gamma P_e - \epsilon (1 - \gamma) P_e - P_e)}{2[1 - \gamma P_e - (1 - \gamma)\epsilon P_e]}
\]

\[
\gamma(1 - P_e) \frac{1 - \gamma P_e - (1 - \gamma)\epsilon P_e}{2[1 - \gamma P_e - (1 - \gamma)\epsilon P_e]}
\]

\[
\frac{(2 - \gamma P_e - (1 - \gamma)\epsilon P_e - P_e) + \epsilon (1 - \gamma)(2 - \gamma P_e - (1 - \gamma)\epsilon P_e - P_e)}{2[1 - \gamma P_e - (1 - \gamma)\epsilon P_e]}
\]

\[
\gamma(1 - P_e) + \frac{1 - \gamma P_e - (1 - \gamma)\epsilon P_e}{2[1 - \gamma P_e - (1 - \gamma)\epsilon P_e]}
\]

\[
\frac{(2 - \gamma P_e - (1 - \gamma)\epsilon P_e - P_e) + \epsilon (1 - \gamma)(2 - \gamma P_e - (1 - \gamma)\epsilon P_e - P_e)}{2[1 - \gamma P_e - (1 - \gamma)\epsilon P_e]}
\]

\[
\gamma P_e[1 - \gamma - \epsilon (1 - \gamma)]
\]

\[
\frac{2[1 - \gamma P_e - (1 - \gamma)\epsilon P_e]}{2[1 - \gamma P_e - (1 - \gamma)\epsilon P_e]}
\]

\[
\gamma P_e[1 - \gamma - \epsilon (1 - \gamma)] > 0.
\]

This completes the characterization of our candidate equilibrium. Now it is possible to establish the proof of proposition 6.

**Proof of Proposition 6**

Assume in negation that only \( S \) names are traded and are bought only by \( G \) types. If \( M \) names are not traded, then \( \text{Pr}(G|M) = 1 \), which follows because such a history can belong only to a good type. This is true because no trade of \( M \) names implies that such a history must belong to a seller who bought an \( S \) name and then failed, and by assumption such a seller must be good. In this case \( w(M) = w(N) \), which in turn implies that \( M \) names must be traded. Thus assume that \( w(M) = w(N) \) and that \( M \) names are traded in equilibrium. The utility that a young good type entering the economy gets out of owning an \( S \) name is

\[
u_g(S) = w(S) + P_e w(S) + (1 - P_e) w(M) + P_e^2 v(S),\]

whereas his utility from not owning a name is

\[
u_g(N) = w(N) + P_e w(S) + (1 - P_e) w(N) + P_e^2 v(S),\]

because after failing in his first period, such a seller who started with a new name will wish to change his bad (\( F \)) name. The benefit from owning a name is therefore

\[
u_g(S) - \nu_g(N) = w(S) - w(N) + (1 - P_e)[w(M) - w(N)].\]

Given that the choice of an opportunistic type depends only on \( w(S) - w(N) \) and not on the name he actually has, the preferences (and, thus, utility differences) of opportunistic types who in equilibrium choose \( e < 1 \) are derived similarly to the calculations above. The utility that a young opportunistic type entering the economy gets out of owning an \( S \) name is

\[
u_o(S) = w(S) + e P_e w(S) + (1 - e P_e) w(M) + (e P_e)^2 v(S) - 2 e(v).\]
whereas his utility from not owning a name is
\[ u_s(N) = w(N) + eP_N w(S) + (1 - eP_s) w(M) + (eP_s)^2 w(S) - 2\varepsilon(e) , \]
and
\[ u_o(S) - u_s(N) = w(S) - w(N) + (1 - eP_s)[w(M) - w(N)] . \]
But since \( w(M) > w(N) \) and \( P_s > eP_s \), \( u_o(S) - u_s(N) > u_o(S) - u_s(N) \). That is, \( O \) types have a larger benefit from owning an \( S \) name, which contradicts the assumption that only \( G \) types buy \( S \) names at \( t = 2 \) in equilibrium. Q.E.D.

References